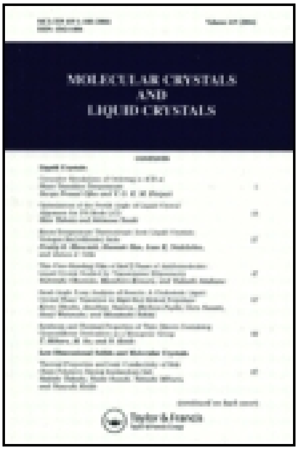


Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

A Change of Structure and Discrete Soliton at Smectic in an Electric Field

Yu Sugimoto [a], Masashi Torikai [a] & Mamoru Yamashita [a]

[a] Department of Physics Engineering, Mie University, Kurimamachiya, Japan

Version of record first published: 05 Oct 2009

Mol. Cryst. Liq. Cryst., Vol. 511, pp. 348/[1818]–359/[1829], 2009

ISSN: 1542-1406 print/1563-5287 online
DOI: 10.1080/15421400903054451

A Change of Structure and Discrete Soliton at Smectic C_α^* in an Electric Field

Yu Sugimoto*, **Masashi Torikai, and Mamoru Yamashita**
Department of Physics Engineering, Mie University,
Kurimamachiya, Japan

Phase transition from chiral smectic C_α phase (SmC_α^*) to smectic C phase (SmC) driven by an electric field is studied from a standpoint of discrete soliton concept, while similar phenomenon occurring at a transition from chiral smectic C phase (SmC^*) to SmC is characterized by a condensation of solitons of sine Gordon equation. As a pitch of SmC_α^* without the field is very short, an equation describing a change of a helical structure is difference equation and the soliton excited at the phase transition should be a discrete type. It is shown that for the pitch larger than three layers, soliton density, which is identical to a wave number of the structure, decreases to zero making a devil's staircase as the field is increased. Though the transition is a continuous type and an interaction between discrete solitons is repulsive like the case of SmC^*, a range of the interaction is shorter than the one for the continuous solitons. On the other hand, for the pitch smaller than three, the wave number increases reaching a bi-layer structure as the field is increased, and finally at a critical field the bi-layer structure changes to the uniform SmC continuously. Three-layer structure is proved to be marginal at the SmC_α^*-SmC transition.

Keywords: devil's staircase; discrete soliton; interaction between solitons; SmC_α^*-SmC transition; soliton excitation

1. INTRODUCTION

Chiral smectic C_α phase (SmC_α^*) appearing in the low temperature side of smectic A phase is ferroelectric and has a helical structure [1–3]. In an electric field, the helical structure of SmC_α^* is unwound and at a certain field strength the phase changes to uniform smectic C phase (SmC) [3–5]. The unwinding mechanism looks similar to the one at a

*Present Affiliation: Nitto Denko Corporation.
Address correspondence to Mamoru Yamashita, Department of Physics Engineering, Mie University, Kurimamachiya 1577, Tsu 514-8507, Japan. E-mail: mayam@phen.mie-u.ac.jp

transition of chiral smectic C phase (SmC*) to SmC [6,7]. In case of SmC*, a helical pitch is quite large and the phase is described in the framework of continuum theory. An equation for an azimuthal angle determining tilt direction (or, polarization direction) is so-called sine-Gordon equation, and the transition is interpreted as a soliton condensation of the sine-Gordon equation [8,9]. On the other hand in SmC$^*_\alpha$, the pitch is very short [10,11], and so a discrete description is required. Then, it is interesting how the nature of the transition to SmC changes and whether the soliton concept is applicable or not. In the previous note, we reported a preliminary study [12]. Here, we show detailed results under various conditions.

The present study is carried out under a condition of constant tilt angle. A free energy of Ginzburg-Landau type is introduced as a natural generalization of the one in the continuum theory for SmC* [12,13], where an important parameter characterizing SmC$^*_\alpha$, δ, is a wave number at zero field, i.e., an inverse of the pitch multiplied by 2π. An equilibrium condition for the azimuthal angle is given by a difference equation, which is solved numerically. It is shown that the soliton concept is useful to understand the transition for $\delta/2\pi < 1/3$, while it is not irrelevant for $\delta/2\pi > 1/3$, that is, quite short pitch.

In the next section, SmC*-SmC transition is reviewed briefly for an explanation of the soliton condensation and a formalism for SmC$^*_\alpha$ is given. The section 3 is devoted to results derived mainly by numerical analysis, and summary is given in section 4.

2. SOLITON CONCEPT AT SMC* AND FORMALISMS OF SMC$^*_\alpha$ IN THE ELECTRIC FIELD

First, the unwinding process of SmC* by the electric field is reviewed [8,9]. Under the condition of constant tilt angle $\theta = \theta_0$, the Ginzburg-Landau free energy is given with the electric field $\boldsymbol{E}$ in x-axis by [13]

$$F = \int \left[\frac{1}{2}k\theta_0^2\left(\frac{d\varphi}{dz} - q_0\right)^2 - E\theta_0 \cos\varphi\right] dz, \tag{1}$$

where φ ($=\varphi(z)$) denotes an azimuthal angle describing a direction of electric polarization, to which the tilt c-director makes a right angle, q_0 a wave number at vanishing field, k an elastic constant and a magnitude of the polarization is taken to be unity for simplicity. Euler-Lagrange equation is derived from Eq. (1) as

$$\frac{d^2\varphi}{dz^2} = \frac{E}{k\theta_0}\sin\varphi, \tag{2}$$

which is nothing but the static sine-Gordon equation. A periodic solution to Eq. (2) is given by

$$\varphi = 2\sin^{-1}\mathrm{sn}\left(\frac{1}{\kappa}\sqrt{\frac{E}{k\theta_0}}(z-z_0);\ \kappa\right)+\pi, \tag{3}$$

in which $\mathrm{sn}(x;\kappa)$ denotes a Jacobi's sn-function with κ, a modulus. The state described by Eq. (3) is called a soliton lattice [8,9]. A wave number of the structure is derived as

$$q = \frac{\pi^2}{4}\frac{q_0}{K(\kappa)E(\kappa)}, \tag{4}$$

where $K(\kappa)$ and $E(\kappa)$ are complete elliptic integrals of the first and second kinds, respectively. From the minimum condition of the free energy (1), the modulus κ is determined by the following equation,

$$\frac{\kappa}{E(\kappa)} = \frac{4}{\pi}\sqrt{\frac{E}{k\theta_0 q_0^2}}. \tag{5}$$

Thus, q is given as a function of E by the use of Eq. (5), showing a continuous curve (*vide* Fig. 2). At a certain field strength, corresponding to the limit $\kappa=1$, q vanishes in Eq. (4), indicating the transition point, and from Eq. (5) a critical field E_c is obtained as

$$E_c = \frac{\pi^2}{16}k\theta_0 q_0^2. \tag{6}$$

We have also one-soliton solution (one-kink) to Eq. (2), and an excitation energy of the soliton is shown to vanish at $E=E_c$, while it is negative for $E<E_c$ [8,9]. Accordingly, many solitons are to be excited at $E<E_c$. However, due to a repulsive interaction between solitons, a density of soliton, which is equivalent to q, remains finite and the soliton lattice structure above-mentioned is achieved. Thus, the transition between SmC* and SmC is interpreted as soliton (or, kink) condensation [8,9].

To describe SmC_α^* in the field, the free energy (1) is discretized in the form given by [12]

$$F = -\sum_i [K\theta_{i+1}\theta_i\cos(\varphi_{i+1}-\varphi_i-\delta)+E\theta_i\cos\varphi_i], \tag{7}$$

for i, a layer number, and for δ, the wave number at vanishing field in a scale unit of layer thickness d. The condition of constant tilt angle $\theta_i = \theta_0$ is also adopted. This type of free energy is already utilized at an unwinding of helical structure of antiferroelectric smectic C phase [14].

Minimum condition of F leads to the equation,

$$\sin(\varphi_{i+1} - \varphi_i - \delta) - \sin(\varphi_i - \varphi_{i-1} - \delta) - e \ \sin\varphi_i = 0, \tag{8}$$

where $e = E/K\theta_0$. Thus, Eq. (8) is interpreted as a discretized form of the sine-Gordon equation (2). Because it is difficult to solve a difference equation generally, we analyze Eq. (8) numerically. A periodic solutions to Eq. (8) is of the form $\varphi_{i\pm p} = \varphi_i + 2\pi m$, ($i = 1, 2, \ldots, p$, $m = 1, 2, \ldots (<p)$) with period p in the scale d, and the wave number q is given by $2\pi m/p$. At the previous preliminary report, the case of $\delta/2\pi = 1/4$ was tested briefly [12]. In the present investigation, cases with various values of δ are studied in detail.

3. NUMERICAL RESULTS

First, the case $\delta/2\pi = 1/4$ is studied. As an example of the periodic solution, a structure with the wave number $q/2\pi = 1/15$ is shown in Figure 1. Such periodic structure is also called a soliton lattice after the terminology in sine-Gordon system. In practice, we see in Figure 1 three discrete solitons located with equal separation, 15

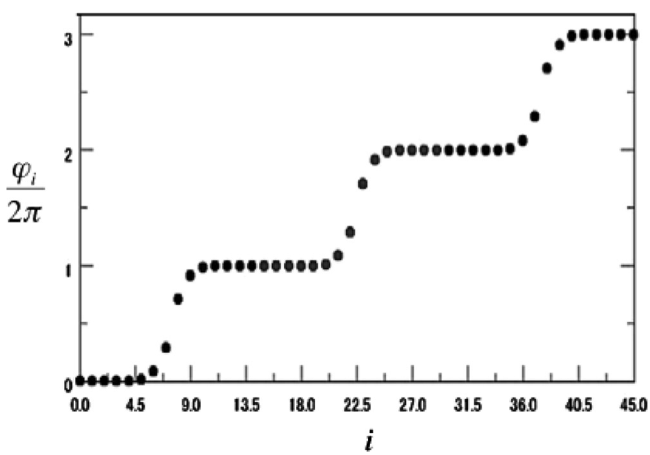


FIGURE 1 Soliton lattice with the period 15.

layers. Apparently, soliton density is equal to $q/2\pi$. For periodic solutions with various wave numbers, free energies per layer are calculated. From the minimum condition of free energy (7), an equilibrium state is chosen, from which e-dependence of the wave number q is obtained as shown in Fig. 2 (a) together with an enlarged figure near $q/2\pi = 3/14$ in (b). A solid curve shown in (a) is the q-e relation for the continuous case given by a combination of Eqs. (4) and (5). The enlarged one in (b) looks like the original whole feature (a), and the q-e relation is considered to be a devil's staircase as characterized by statistical self-similarity [15]. We observe in Figure 2 (a) that q decreases to zero as e approaches to e_c (=1.231592), and the transition from SmC$^*_\alpha$ to SmC is concluded to be continuous. (Practically, we have calculated the phases with wave numbers up to $2\pi/15$ near e_c). Accordingly, the interaction between solitons is repulsive. Though these properties are similar to the corresponding ones at SmC* with continuum description, an interaction range for discrete solitons is shorter than the one between continuous solitons, because of a sharp increase of q from zero just below e_c in comparison with the increase for the continuous case. Moreover, the q-e relations are different even qualitatively; in SmC$^*_\alpha$ the devil's staircase structure is observed, while continuous change occurs in SmC*.

To certify whether the q-e relation is truly the devil's staircase or not, it is required to prove an existence of finite stable interval of the field for any wave number. Here, we show that the stable interval is finite for $q/2\pi = 1/5$ as a typical but arbitrary one. Wave numbers, $q/2\pi$, of phase series converging to the phase with $q/2\pi = 1/5$ are chosen in Farey series $n/(5n-1)$ and $n/(5n+1)$ with $n = 2, 3, 4, \ldots$

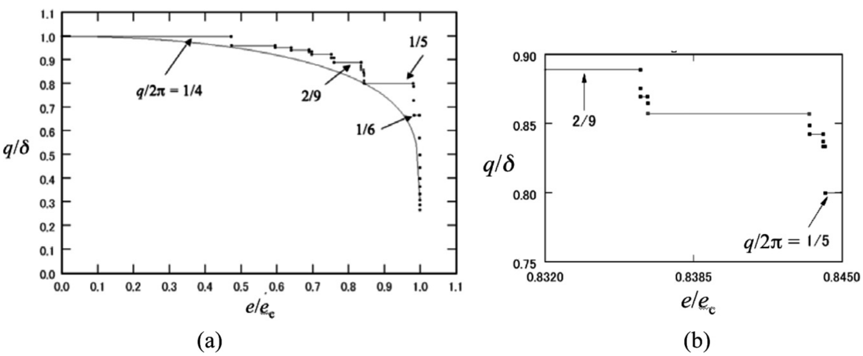
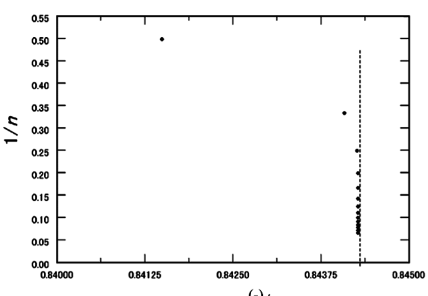


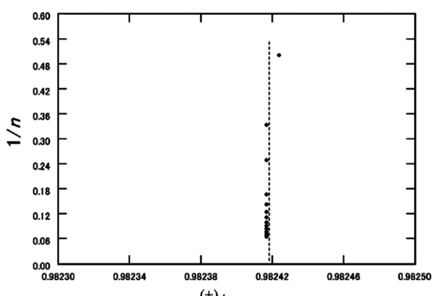
FIGURE 2 Field dependence of the wave number at $\delta/2\pi = 1/4$ (a) and enlarged one near $\square/2\pi = 3/14$ (b).

It is shown how coexisting point between the phases with $q/2\pi = 1/5$ and $n/(5n-1)$ changes as n is increased in Figure 3(a) and the one between $q/2\pi = 1/5$ and $n/(5n+1)$ in Figure 3(b), respectively. Apparently, values of e/e_c at the coexisting points, $\mathrm{e}_\mathrm{n}^{(-)}/\mathrm{e}_\mathrm{c}$ and $\mathrm{e}_\mathrm{n}^{(+)}/\mathrm{e}_\mathrm{c}$, converge to $\mathrm{e}_\mathrm{c}^{(-)}/\mathrm{e}_\mathrm{c}$ (=0.844293) and $\mathrm{e}_\mathrm{c}^{(+)}/\mathrm{e}_\mathrm{c}$ (=0.982417), respectively. Thus, the phase with $q/2\pi = 1/5$ is proved to be stable in an interval $\mathrm{e}_\mathrm{c}^{(-)} < \mathrm{e} < \mathrm{e}_\mathrm{c}^{(+)}$. We can show that any phase has a finite stable interval within a limit of numerical precision.

Here, sharp convergences of $\mathrm{e}_\mathrm{n}^{(-)}/\mathrm{e}_\mathrm{c}$ and $\mathrm{e}_\mathrm{n}^{(+)}/\mathrm{e}_\mathrm{c}$ in Figure 3 are noticed. Roughly speaking, a structure of phase with $q/2\pi = n/(5n-1)$ is composed of $(n-1)$ 5-layer blocks and one 4-layer block. In other words, one compressed-soliton (i.e., 4-layer block) is inserted per $5(n-1)$ layers in the structure $q/2\pi = 1/5$. Thus, the structure is interpreted as a compressed-soliton lattice with a period $(5n-1)$ [16]. In the limit, $n \to \infty$, a separation between compressed-solitons diverges and the structure is reduced to a one-compressed-soliton state excited at the phase with $q/2\pi = 1/5$. At $\mathrm{e} = \mathrm{e}_\mathrm{c}^{(-)}$, the free energy for the one-compressed-soliton state agrees with the one for $q/2\pi = 1/5$, which means that an excitation energy of the compressed-soliton vanishes. The sharp increase of $1/\mathrm{n} - \mathrm{e}_\mathrm{n}^{(-)}$ relation just below $\mathrm{e}_\mathrm{c}^{(-)}$ in Figure 3 (a), or, insensitivity of $\mathrm{e}_\mathrm{n}^{(-)}$ to n, indicates a short range character of an interaction between the compressed-solitons. Analogous argument is possible for the series of phases, $n/(5n+1)$, where we have only to replace the compressed-soliton by a stretched-soliton (6-layer block). The shortness of the interaction range of compressed-solitons (and stretched-solitons) reminds us of the short range interaction between the discrete solitons above-mentioned. Hence, we conclude that the



(a) Lower limit



(b) Upper limit

FIGURE 3 Convergence of edges showing stable interval of the phase, $q/2\pi = 1/5$.

shortness of the interaction range for solitons is attributed to the discreteness of the system.

We have carried out similar analysis for the cases, $\delta/2\pi = 1/5$, 1/7, 1/10 and 2/7. For each case, we obtain the devil's staircase structure and behaviour is same qualitatively to the case of $\delta/2\pi = 1/4$. The q–e relations for $\delta/2\pi = 1/4$, 1/5, 1/10 are shown together with the continuous case in Figure 4. A natural approach of the q–e relation to the continuous curve is observed as δ is decreased, keeping the devil's staircase structure.

To sum up the results for the cases, $\delta/2\pi < 1/3$, the soliton concept is applicable to SmC_α^*-SmC transition; the transition is continuous and the interaction between discrete solitons is repulsive, but the interaction range is shorter than the one at the continuous case, due to the discreteness of the system. In addition to the difference at the interaction range, the q–e relation makes a devil's staircase structure in contrast with the continuous change at SmC* described by the continuum theory.

Next, we proceed to studying the case of quite short pitch, $1/3 < \delta/2\pi < 1/2$, where values, 4/11, 3/8, 2/5 and 3/7 are actually taken for $\delta/2\pi$. In any case, the phase changes to bi-layer structure ($q/2\pi = 1/2$) before reaching e_c, and the bi-layer structure approaches continuously to SmC at e_c. Truly, we can prove this continuous

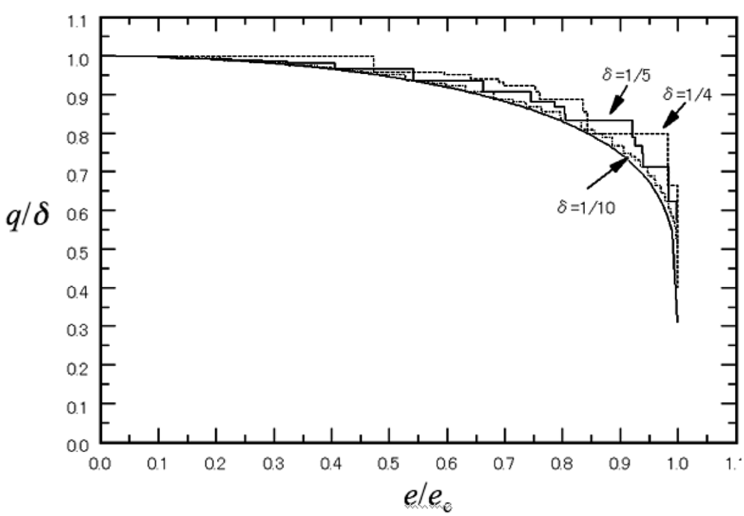


FIGURE 4 Field dependence of the wave number at $\delta/2\pi = 1/4$, 1/5, 1/10 together with the continuous one.

transition analytically. At the bi-layer structure, we assign $\varphi_{2k} = \varphi$ and $\varphi_{2k+1} = -\varphi$. Then, following solution to Eq. (8) is derived,

$$\varphi = \cos^{-1}\left(-\frac{e}{4\cos\delta}\right). \tag{9}$$

For increasing *e,* φ decreases in Eq. (9) and vanishes at a critical field, e_c, given by

$$e_c = -4\cos\delta. \tag{10}$$

Free energy per layer, $f_{1/2}$, is calculated from Eq. (7) together with Eq. (9) as $f_{1/2} = \cos\delta + e^2/(8\cos\delta)$, which is compared with the one at SmC, $f_C = -\cos\delta - e$,

$$f_{1/2} - f_C = -\frac{e}{2}\left(\sqrt{\frac{e}{-4\cos\delta}} - \sqrt{\frac{-4\cos\delta}{e}}\right)^2 \leq 0. \tag{11}$$

The equality in Eq. (11) is satisfied only at $e = e_c$, where φ vanishes. We show e-dependence of q in Figure 5, in which intermediate phases are observed apparently at the case, $\delta/2\pi = 3/7$; wave numbers of the intermediate phases, $q/2\pi$, are 7/16, 4/9, 5/11, i.e., a fraction of Farey series composed from 3/7 and 1/2. Possibly, q-e relation makes a devil's staircase in an interval, $\delta/2\pi < q/2\pi < 1/2$. However, because of the restriction of precision of numerical analysis, we could not

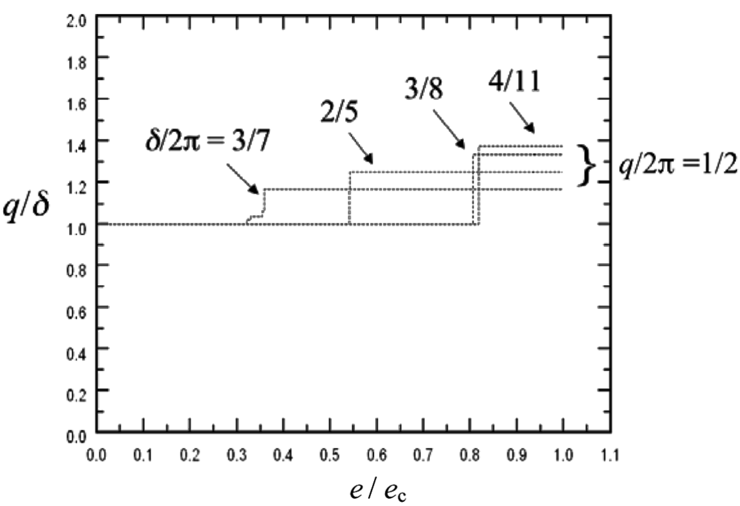


FIGURE 5 The e-dependence of q for $\delta/2\pi = 4/11$, 3/8, 2/5 and 3/7 (>1/3).

certify other phases to exist, and consequently the structure of q–e relation for this case is not clarified yet. We also find an intermediate phase with $q/2\pi = 7/15$ between 2/5 and 1/2 for $\delta/2\pi = 2/5$, while no intermediate phase is found for $\delta/2\pi = 3/8$ and 4/11. So far, the structure of the q–e relation for $\delta/2\pi$ between 1/3 and 1/2 is an open question to be solved in a future. Nevertheless, it is worthwhile to point out that the wave number increases as the field is increased, which is compared with the cases for $\delta/2\pi < 1/3$, where the wave number decreases as the field is increased. Anyway, for the quite short pitch region, $1/3 < \delta/2\pi < 1/2$, the soliton concept is not applicable to the SmC$^*_\alpha$-SmC transition in the present stage.

Finally the case with $\delta/2\pi = 1/3$ is studied. By numerical analysis, a structure of three-layer is shown to be always stable below the critical field, keeping a symmetry, $\varphi_{3k} = 0$, $\varphi_{3k+1} = -\varphi_{3k+2} = \varphi$. At the critical field, $e_c = 2$, discontinuous change to SmC occurs, and φ jumps from $\pi/3$ to zero. A change of free energy per layer, f, is shown for $q/2\pi = 1/3$, together with those for 1/4, 1/2 and 0 (SmC), in Figure 6. All of these curves cross at $e = e_c$ (=2), where the curve for 1/2 is given by $f_{1/2}$ derived in the above and the value, $e_c = 2$, agrees with Eq. (10) for $\delta/2\pi = 1/3$. Figure 6, showing the stability of the phase with $q/2\pi = 1/3$, also indicates that the condition, $\delta = 2\pi/3$ ($=\delta^*$), is a marginal. If δ is decreased from δ^*, then the curve for $q/2\pi = 1/4$ becomes lower than the one for $q/2\pi = 1/3$ near e_c and q–e relation is considered to make a devil's staircase, as stated above, with wave number from δ to zero via $q = 2\pi/4$. On the other hand, if δ is increased

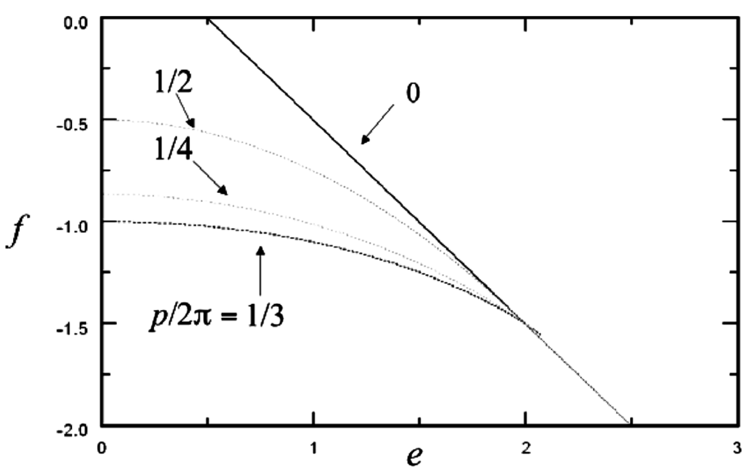


FIGURE 6 Changes of free energies for various wave numbers at $\delta/2\pi = 1/3$.

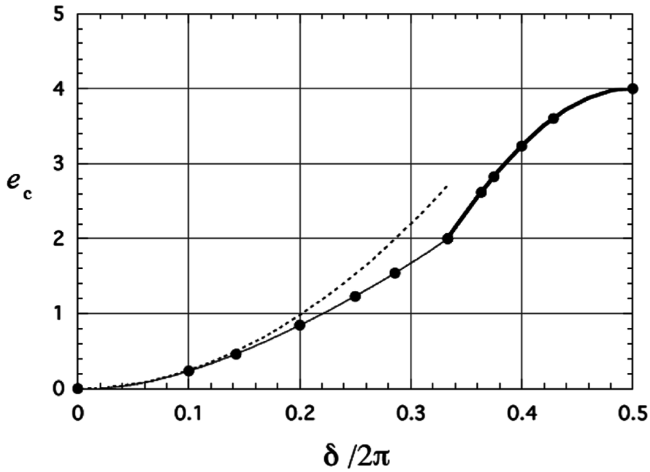


FIGURE 7 Critical field e_c for various δ.

from δ^*, the curve with $q/2\pi = 1/2$ becomes lower than the one for $1/3$ near e_c and the wave number changes in the course as shown in Figure 5 for $\delta > 2\pi/3$.

The marginality of δ ($=\delta^*$) is also observed at a behaviour of e_c for the change of δ, as shown in Figure 7. The bold curve is given by Eq. (10) and the thin curve is drawn as a guide for eye. The dotted curve is an estimate of continuum limit of the present system. In Eq. (7), a continuum approximation is taken, and by comparing this to the free energy of Eq. (1), we obtain following correspondence; $q_0 = \delta/d$ and $E/k\theta_0 = e/d^2$. Then, by using of Eq. (6), we obtain the critical field estimated by continous limit, $e_c^{(c)}$, as

$$e_c^{(c)} = \frac{\pi^2}{16}\delta^2. \tag{12}$$

Agreement of calculated e_c with Eq. (12) at small δ value is a natural consequence, corresponding to the approach of q–e relation to the continuous one as mentioned at Figure 4. Thus, the crossover at δ^* is certified apparently in the e_c–δ relation.

4. SUMMARY

Unwinding process of SmC_α^* to SmC driven by an electric field is studied on the basis of simple phenomenological discrete model, in which an interpretation of a mechanism of the SmC_α^*-SmC transition to be a condensation of discrete solitons is tested. It is certified that the mechanism depends crucially on whether a pitch of $\mathrm{SmC}_{\alpha}^{*}$ without field is larger or smaller than three layers. For the case of pitch larger than three layers, a wave number, which is identical to soliton density, decreases to zero making a devil's staircase as the field is increased. Similar to a continuous case of transition between SmC^{*} and SmC, the transition is a continuous type and an interaction between discrete solitons is repulsive, while a range of the interaction is shorter than the one compared with the continuous solitons. On the other side, for the case of the pitch smaller than three, the wave number increases reaching to a bi-layer structure with increasing field, in which intermediate phases are also found to occur, though a property of the wave number versus field relation is not clarified yet. The bi-layer structure changes to the uniform SmC continuously at a critical field. Thus, the soliton concept seems not to be applicable for this quite short pitch, at least, to $\mathrm{SmC}_{\alpha}^{*}$-SmC transition. The system with period of three layers is shown to be just a watershed, where three-layer structure remains up to the critical field at which the phase changes to SmC discontinuously. Dependence of the critical field on the pitch at zero-field also exhibits the crossover at the period of three layers.

In a representative material MHPOCBC showing $\mathrm{SmC}_{\alpha}^{*}$, it has been reported that $\mathrm{SmC}_{\alpha}^{*}$ looks as if the phase is separated into two different phases appearing in high temperature and low temperature regions, respectively, by applying the electric field [17]. In respect to this, the crossover above-mentioned is possibly related to the separation, i.e., at the temperature the separation of $\mathrm{SmC}_{\alpha}^{*}$ into two phases occurs, the pitch is considered to be of three layers, while the pitch at vanishing field is not clarified experimentally so far in MHPOCBC. On the other hand, something like a staircase structure is observed at an apparent optical axis for increasing electric field [1]. It is also interesting whether the devil's staircase structure derived in this article is applicable to this experimental fact or not. Even though our model for $\mathrm{SmC}_{\alpha}^{*}$ is quite simple compared with realistic models [18,19], we will try to explain related experimental facts, such as the polarization current [2] in addition to the change of optical axis. Anyway, experimental data on temperature dependence of the pitch [10,11,20] are required for various materials.

At the experimental study of optical axis, the change is observed even at the induced SmC for the electric field larger than a critical field [1], showing an increase of the tilt angle. Accordingly, the effect of variable tilt angle is of interest, while the constant tilt condition is applied in the present study. In practice, an interaction between solitons can change to be attractive due to the variable amplitude at SmC*, and eventually the transition between SmC* to SmC turns to a discontinuous one [8,9]. The effect of variable amplitude on discrete soliton is an open question to be attacked in a near future.

REFERENCES

[1] Hiraoka, H., Takanishi, Y., Skarp, K., Takezoe, H., & Fukuda, A. (1991). *Jpn. J. Appl. Phys.*, *30*, L1819.
[2] Takanishi, Y., Takezoe, H., Fukuda, A., Isozaki, T., Suzuki, Y., & Kawamura, I. (1991). *Jpn. J. Appl. Phys.*, *30*, 2023.
[3] Hiraoka, H., Takanishi, Y., Takezoe, H., Fukuda, A., Isozaki, T., Suzuki, Y., & Kawamura, I. (1992). *Jpn. J. Appl. Phys.*, *31*, 3394.
[4] Bourny, V. & Orihara, H. (2001). *Phys. Rev.*, *E63*, 021703.
[5] Orihara, H., Naruse, Y., Yagyu, M., Fajar, A., & Uto, S. (2005). *Phys. Rev.*, *E72*, 040701.
[6] Meyer, R. B., Liebert, L., Strzelecki, L., & Keller, P. (1975). *J. Phys. Lett. France*, *36*, L-69.
[7] Meyer, R. B. (1977). *Mol. Cryst. Liq. Cryst.*, *40*, 33.
[8] Yamashita, M. (1985). *Prog. Theor. Phys.*, *74*, 622.
[9] Yamashita, M. (1991). *Solitons in Liquid Crystals*, Lam, L. & Prost, J. (Eds.), Springer-Verlag: Berlin, Chap. 10.
[10] Mach, P., Pindak, R., Levelut, A.-M., Barois, P., Nguyen, H. T., Huang, C. C., & Furenlid, L. (1998). *Phys. Rev. Lett.*, *81*, 1015.
[11] Mach, P., Pindak, R., Levelut, A.-M., Barois, P., Nguyen, H. T., Baltes, H., Hird, M., Toyne, K., Seed, A., Goodby, J. W., Huang, C. C., & Furenlid, L. (1999). *Phys. Rev. E*, *60*, 6793.
[12] Torikai, M. & Yamashita, M. (2007). *Mol. Cryst. Liq. Cryst.*, *465*, 239.
[13] Pikin, S. A. & Indenbom, V. L. (1978). *Sov. Phys. –Usp.*, *21*, 487.
[14] Parry-Jones, L. A. & Elston, S. J. (2001). *Phys. Rev. E*, *63*, 050701(R).
[15] Bak, P. & Bruinsma, R. (1982). *Phys. Rev. Lett.*, *49*, 249.
[16] Yamashita, M. & Takeno, S. (1999). *J. Phys. Soc. Jpn.* *68*, 1473.
[17] Shtykov, N. M., Chandani, A. D. L., Emelyanenko, A. V., Fukuda, A., & Vij, J. K. (2005). *Phys. Rev. E*, *71*, 021711.
[18] Cepic, M. & Zeks, B. (2007). *Mol. Cryst. Liq. Cryst.*, *236*, 61.
[19] Emelyanenko, A. V., Fukuda, A., & Vij, J. K. (2006). *Phys. Rev. E*, *74*, 011705.
[20] Cady, A., Han, X. F., Olson, D. A., Orihara, H., & Huang, C. C. (2003) *Phys. Rev. Lett.*, *91*, 125502.